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Direct CP-violation in untagged B-meson decays

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Abstract

Direct CP-violation can exist in untagged, neutral B-meson decays to certain self-conjugate, hadronic final states. It can occur if the resonances which appear therein permit the identification of distinct, CP-conjugate states—in analogy to stereochemistry, we term such states “CP-enantiomers”. These states permit the construction of a CP-odd amplitude combination in the untagged decay rate, which is non-zero if direct CP-violation is present. The decay $B \to \pi^+\pi^-\pi^0$, containing the distinct CP-conjugate states $\rho^+\pi^-$ and $\rho^-\pi^+$, provides one such example of a CP-enantiomeric pair. We illustrate the possibilities in various multi-particle final states.

The measurement of a non-zero value of $\mathrm{Re}(\epsilon'/\epsilon)$ in $K \to \pi\pi$ decays establishes the existence of direct CP-violation in nature [1], and provides an important first check of the mechanism of CP-violation in the Standard Model (SM). Numerically, however, $\mathrm{Re}(\epsilon'/\epsilon)$ is very small. In the SM, this results, in part, from the weakness of inter-generational mixing [2]; the associated CP-violating parameter δ_{KM} in the Cabibbo–Kobayashi–Maskawa (CKM) matrix need not be small [3]. Indeed, the measurement of a large CP-asymmetry in $B^0(\overline{B}^0) \to J/\psi K_s$ decay and related modes [4], induced through the interference of B^0–$\overline{B}^0$ mixing and direct decay, suggests that $\delta_{\mathrm{KM}} \sim \mathcal{O}(1)$ [5]. Nevertheless, the observation of direct CP-violation in the B-meson system is needed to clarify the mechanism of CP-violation, to confirm that the Kobayashi–Maskawa (KM) phase drives the CP-violating effects seen. In the SM, direct CP-violation is anticipated to be much larger in B-meson decays than in K-meson decays [6]. The observation of direct CP-violation in B-meson decays would falsify models in which the CP-violating interactions are “essentially” superweak [7,8]. In this Letter, we discuss how the presence of direct CP-violation can be elucidated in untagged B-meson decays—the practical advantage of this strategy is the far larger statistical sample of events available.

The rich resonance structure of the multiparticle ($n \geqslant 2$) final states accessible in heavy meson decays provides the possibility of observing direct CP-violation without tagging the flavor of the decaying, neutral meson. The familiar condition for the presence of direct CP-violation, $|\bar{A}_{\bar{f}}/A_f| \neq 1$, can be met by a non-zero value of the partial rate asymme-

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try, so that, seemingly, one would want to distinguish empirically a decay with amplitude A_f from that of its CP-conjugate mode with amplitude $\bar{A}_{\bar{f}}$. However, in neutral B, D-meson decays to self-conjugate final states [9–11], direct CP-violation in untagged decays may nevertheless occur. It can occur if we can separate the self-conjugate final state, via the resonances which appear, into distinct, CP-conjugate states. This condition finds it analogue in stereochemistry: we refer to molecules which are non-superimposable, mirror images of each other as enantiomers [12]. Accordingly, we refer to non-superimposable, CP-conjugate states as *CP enantiomers*. In $B \to \pi^+\pi^-\pi^0$ decay, e.g., the intermediate states $\rho^+\pi^-$ and $\rho^-\pi^+$ form CP enantiomers, as they are distinct, CP-conjugate states. As a result, the untagged decay rate contains a CP-odd amplitude combination. The empirical presence of this CP-odd interference term in the untagged decay rate would be realized in the Dalitz plot as a population asymmetry, reflective of direct CP-violation.

We shall use $B \to \pi^+\pi^-\pi^0$ decay as a paradigm of how direct CP-violation can occur in untagged B-meson decays. In what follows, we shall largely follow the notation and conventions of Quinn and Silva [13]. Consider the amplitudes for $B^0(\overline{B}^0) \to \pi^+\pi^-\pi^0$ decay:

$$\begin{aligned}
&A\big(B^0(p_B) \to \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)\big)\\
&\quad = f_+[u]a_{+-} + f_-[s]a_{-+} + f_0[t]a_{00},\\
&\bar{A}\big(\overline{B}^0(p_B) \to \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)\big)\\
&\quad = f_+[u]\bar{a}_{+-} + f_-[s]\bar{a}_{-+} + f_0[t]\bar{a}_{00},
\end{aligned} \tag{1}$$

where the two-body decay amplitudes are given by $a_{+-} = A(B^0 \to \rho^+\pi^-)$, $a_{-+} = A(B^0 \to \rho^-\pi^+)$, and $a_{00} = A(B^0 \to \rho^0\pi^0)$ and f_i is the form factor describing $\rho^i \to \pi\pi$. We have used $s = (p_- + p_0)^2$, $t = (p_+ + p_-)^2$, and $u = (p_+ + p_0)^2$.[2] For clarity, note that $\bar{a}_{+-} = \bar{A}(\overline{B}^0 \to \rho^+\pi^-)$ and $\bar{a}_{-+} = \bar{A}(\overline{B}^0 \to \rho^-\pi^+)$. Since $\rho^+\pi^-$ and $\rho^-\pi^+$ are distinct, CP-conjugate states, the amplitudes $a_g = a_{+-} + a_{-+}$ and $a_u = a_{+-} - a_{-+}$ have distinct properties under CP. That is, if we define $\bar{a}_g = \bar{a}_{+-} + \bar{a}_{-+}$ and $\bar{a}_u = \bar{a}_{+-} - \bar{a}_{-+}$, we see, under an appropriate choice of phase conventions, that the CP conjugate of a_g is $\bar{a}_g$, whereas the CP conjugate of a_u is $-\bar{a}_u$. With $a_n = 2a_{00}$ we have

$$\begin{aligned}
A_{3\pi} &\equiv A\big(B^0 \to \pi^+\pi^-\pi^0\big)\\
&= f_g[u,s]a_g + f_u[u,s]a_u + f_n[t]a_n,\\
\bar{A}_{3\pi} &\equiv \bar{A}\big(\overline{B}^0 \to \pi^+\pi^-\pi^0\big)\\
&= f_g[u,s]\bar{a}_g + f_u[u,s]\bar{a}_u + f_n[t]\bar{a}_n,
\end{aligned} \tag{2}$$

where $f_g[u,s] = (f_+[u] + f_-[s])/2$, $f_u[u,s] = (f_+[u] - f_-[s])/2$, and $f_n[t] = f_0[t]/2$. Neglecting the width difference of the B-meson mass eigenstates, as $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$ and $|\Delta\Gamma| \ll \Gamma \equiv (\Gamma_H + \Gamma_L)/2$, the decay rate into $\pi^+\pi^-\pi^0$ for a B^0 meson at time $t = 0$ is given by [15]

$$\begin{aligned}
&\Gamma\big(B^0(t) \to \pi^+\pi^-\pi^0\big)\\
&\quad = |A_{3\pi}|^2 e^{-\Gamma t}\bigg[\frac{1+|\lambda_{3\pi}|^2}{2} + \frac{1-|\lambda_{3\pi}|^2}{2}\cos(\Delta m t)\\
&\qquad - \operatorname{Im}\lambda_{3\pi}\sin(\Delta m t)\bigg],
\end{aligned} \tag{3}$$

whereas the analogous decay rate for a $\overline{B}^0$ meson at time $t = 0$ is given by

$$\begin{aligned}
&\Gamma\big(\overline{B}^0(t) \to \pi^+\pi^-\pi^0\big)\\
&\quad = |A_{3\pi}|^2 e^{-\Gamma t}\bigg[\frac{1+|\lambda_{3\pi}|^2}{2} - \frac{1-|\lambda_{3\pi}|^2}{2}\cos(\Delta m t)\\
&\qquad + \operatorname{Im}\lambda_{3\pi}\sin(\Delta m t)\bigg].
\end{aligned} \tag{4}$$

Note that $\lambda_{3\pi} \equiv q\bar{A}_{3\pi}/pA_{3\pi}$ and $\Delta m \equiv M_H - M_L$. We neglect $\Delta\Gamma$, so that we set $|q/p| = 1$. Untagged observables, for which the identity of the B meson at $t = 0$ is unimportant, correspond to $\Gamma(B^0(t) \to \pi^+\pi^-\pi^0) + \Gamma(\overline{B}^0(t) \to \pi^+\pi^-\pi^0) \propto |A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2$. We have

[2] We have implicitly summed over the ρ^i polarization. Defining $\langle\pi^0(p_0)\pi^-(p_-)|\rho^-(p_\rho,\epsilon)\rangle \equiv -g_\rho\epsilon\cdot(p_0 - p_-)$ and $\langle\rho^i(\epsilon,p_\rho)\pi^j(p_\pi)|\mathcal{H}_{\rm eff}|B^0(p_B)\rangle \equiv 2\epsilon^*\cdot p_\pi a_{ij}$, where $\mathcal{H}_{\rm eff}$ is the $|\Delta B| = 1$ effective Hamiltonian, we find $A(B^0(p_B) \to \pi^+(p_+)\pi^-(p_-)\pi^0(p_0)) = a^{00}(s-u)F_0(t) + a^{+-}(t-s)F_+(u) + a^{-+}(u-t)F_-(s)$, where the pions' masses are given by $M_{\pi^\pm} = M_{\pi^0} = M_\pi$. The form factor $F_i(x)$ can be described by a Breit–Wigner form $g_\rho/(x - M_\rho^2 + i\Gamma_\rho M_\rho)$, or a more sophisticated function, consistent with the theoretical constraints of analyticity, time-reversal-invariance, and unitarity, see Ref. [14] for all details. Note that, e.g., $f_+[u] \equiv (t-s)F_+(u)$.

$$|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 = \sum_i \big(|a_i|^2 + |\bar{a}_i|^2\big)|f_i|^2 + 2\sum_{i<j}\big[\mathrm{Re}\big(f_i f_j^*\big)\,\mathrm{Re}\big(a_i a_j^* + \bar{a}_i\bar{a}_j^*\big) - \mathrm{Im}\big(f_i f_j^*\big)\,\mathrm{Im}\big(a_i a_j^* + \bar{a}_i\bar{a}_j^*\big)\big], \qquad (5)$$

where $i, j \in g, u, n$, noting that i, j labels are not repeated in the sum labelled "$i < j$". The different products $f_i f_j^*$ are distinguishable through the Dalitz plot of this decay, so that the coefficients of these functions are empirically distinct [13]. For our purposes the crucial point is that these observables, as first noted by Quinn and Silva [13], can be of CP-odd character. In particular, the presence of

$$a_g a_u^* + \bar{a}_g\bar{a}_u^* \quad \text{and/or} \quad a_n a_u^* + \bar{a}_n\bar{a}_u^* \qquad (6)$$

is reflective of direct CP-violation. Physically these observables correspond to a population asymmetry under the exchange of u and s (or of p_+ and p_-) across the Dalitz plot. To make the geometric sense of this construction clear, consider a Dalitz plot in u versus s, that is, in the invariant masses of the $\pi^+\pi^0$ and $\pi^-\pi^0$ pairs, respectively—such a plot is shown in Fig. 1 of Ref. [16]. The presence of the CP-odd amplitude $a_g a_u^* + \bar{a}_g\bar{a}_u^*$, e.g., engenders a population asymmetry about the $u = s$ "mirror line"; specifically, the number of charged ρ events in the $u > s$ region differs from that in the $s < u$ region. Note that the functional form of $f_+(u)$ and $f_+(s)$ restrict the product $f_g f_u$ to the $\rho^\pm$ bands in the Dalitz plot. The asymmetry is largest in the regions where the ρ^i bands overlap, though the restricted number of events in the overlap region make it more efficacious to compare the entire population of the charged ρ bands in the $u > s$ and $u < s$ regions [17]. The second amplitude combination of Eq. (6) is determined by the population asymmetry across the $u = s$ line in the regions in which the $\rho^\pm$ and ρ^0 bands overlap. A population asymmetry in $B, \overline{B} \to \pi^+\pi^-\pi^0$ decay about the $u = s$ line is also a signature of direct CP-violation. However, non-zero values of the amplitude combinations of Eq. (6) do not guarantee its existence as cancellations, though likely incomplete, can occur. The direct CP-violating observables of Eq. (6) can persist even if the strong phases of the a_j amplitudes were zero. To illustrate, we parametrize $a_j = T_j \exp(-i\alpha) + P_j$ and $P_j/T_j = r_j \exp(i\delta_j)$, where $r_j > 0$ and δ_j is the strong phase of interest.[3] Thus

$$a_g a_u^* + \bar{a}_g\bar{a}_u^* = -2T_g T_u^* \sin\alpha\big[r_g \sin\delta_g + r_u \sin\delta_u - i(r_g\cos\delta_g - r_u\cos\delta_u)\big]. \qquad (7)$$

The real and imaginary parts of this relation are each observable, as they correspond to distinct f_i-dependent terms in Eq. (5). The combination $T_g T_u^*$ can be complex, though we assume it to be real for crispness of discussion. In the imaginary part, we see that direct CP-violation can exist if the strong phases of a_j vanish, i.e., if $\delta_u = \delta_g = 0$; merely the difference of r_g and r_u must be non-zero to realize direct CP-violation were $\sin\alpha \neq 0$. If $\delta_j = 0$ the strong phase is provided by the resonance width, $\mathrm{Im}(f_i f_j^*) \neq 0$. Theoretical estimates suggest that r_g and r_u are both non-zero and unequal [18]. In contrast, a partial rate asymmetry can be written as

$$|a_g|^2 - |\bar{a}_g|^2 = -4|T_g|^2 r_g \sin\delta_g \sin\alpha, \qquad (8)$$

yielding the familiar result that both r_g and δ_g must be non-zero to yield direct CP-violation were $\sin\alpha \neq 0$. Such conditions are realized in the real part of Eq. (7) as well, so that the direct CP-violating observables we propose can be manifest irrespective of the strong phases of a_j, as they can be non-zero were δ_j either zero or 90 degrees. This greater flexibility arises as the combination $P_g/T_g - P_u^*/T_u^*$ appears in Eq. (7), whereas $P_g/T_g - P_g^*/T_g^*$, e.g., appears in the partial rate asymmetry.

Interestingly, similar considerations arise in the angular analysis of $B \to V_1 V_2$ decays: there, too, a CP-odd interference term can beget direct CP-violation in untagged decays [19,20]. There are three helicity amplitudes, labelled by the helicity $\lambda \in (0, \pm 1)$ of either vector meson in $B \to V_1 V_2$ decay. Working in a transversity basis [21], we can define the amplitudes $A_\parallel \equiv (A_{+1} + A_{-1})/\sqrt{2}$ and $A_\perp \equiv (A_{+1} - A_{-1})/\sqrt{2}$ [22]. The full angular distribution of the summed amplitudes for B^0 and $\overline{B}^0$ decay permits the

[3] We drop an overall factor of $\exp(-i\beta)$ in a_j as it is of no consequence to our discussion.

extraction of the imaginary part of the amplitude combinations of Eq. (6), under the identification $a_g \to A_\parallel$, $a_u \to A_\perp$, and $a_n \to A_0$. Moreover, these untagged contributions are insensitive to the strong phase [23].

The conditions which permit the realization of direct CP-violation in untagged modes are quite general. We need only consider self-conjugate final states whose resonances encode enantiomeric pair correlations. Self-conjugate final states can be realized not only through the $b \to dq\bar{q}$ decays of B_d mesons but also through the $b \to sq\bar{q}$ decays of B_s mesons, where $q \in u, d, s, c$ quarks. The KM picture of CP-violation suggests that direct CP-violating effects ought be suppressed by a factor of $\mathcal{O}(\lambda^2) \sim 1/20$ in B_s meson decay to charmed, self-conjugate states. Thus the goals of direct CP-violation searches in B_d and B_s meson decays can be distinct. The appearance of direct CP-violation in B_d-meson decays would substantiate the KM picture of CP-violation, whereas its appearance in any significant measure in B_s decays to charmed final states would signal the presence of new physics. Physics with B_s mesons is important to the future B-physics programs at the Tevatron [24] and at the LHC [25]. The effective tagging efficiency $\epsilon_{\rm eff}$ is significantly smaller in a hadronic environment, cf. $\epsilon_{\rm eff} \sim 7\%$ [26] with $\epsilon_{\rm eff} \sim 27\%$ [27,28] at the B-factories, so that the untagged studies we propose significantly enable direct CP-violation searches at these facilities.[4]

Let us enumerate three-, four-, and five-particle final states in B_d decay which could yield direct CP-violation in the KM picture. We thus focus on $b \to du\bar{u}$ and $b \to dc\bar{c}$ decays, and some possibilities are given in Table 1—we do not attempt to be exhaustive. The CP-enantiomers are useful in the sense we have illustrated in $B \to \rho\pi$ decay: they permit the formation of manifestly CP-odd amplitude combinations which can be probed through asymmetries in the population of events in the regions where the resonances of the CP-enantiomeric pair occur. We expect the CP-violating effects to be larger for broad resonances such as the ρ and $K^*(892)$. Note that the final states $K^+K^-\pi^0$ and $K^+K^-\pi^+\pi^-$, with the CP enantiomers indicated, also lend themselves to direct CP-violation searches in B_s decay. Multiparticle final states can support more than one CP-enantiomeric pair, as illustrated in $B_d \to \pi^+\pi^-\pi^+\pi^-\pi^0$ decay. In the case of CP enantiomers which have more than one spin one particle, as in $(a_1(1260)^+\rho^-, a_1(1260)^-\rho^+)$, or which are not realized by a quasi-two-body decay, as in $(\rho^+\pi^-\pi^+\pi^-, \rho^-\pi^+\pi^+\pi^-)$, a caution is in order. For example, the presence of two spin-one particles in the final state implies that partial waves with $L = 0, 1$, or 2 can occur; the factor $(-1)^L$ impacts the CP of the state. The sum and difference of the amplitudes associated with $B^0 \to a_1(1260)^+\rho^-$ and $\overline{B}^0 \to a_1(1260)^-\rho^+$ decay still yield combinations with definite CP properties for any particular L, but for $L = 0$ or 2 the sum of amplitudes, with a suitable choice of phase conventions, does not change sign under CP, whereas for $L = 1$ the sum of amplitudes do change sign under CP. In either event, for fixed L, the CP-odd amplitude combination of Eq. (7) appears and drives a population asymmetry under the exchange of the momentum of a π^+ emerging from the $a_1(1260)^+$ and that of the π^- from the ρ^- in the region of the Dalitz plot where the resonances of the CP-enantiomeric pair occur. States of fixed L can be realized through a helicity analysis; the formation of the $A_\perp$ amplitude, e.g., selects the $L = 1$ state [21]. In the absence of a helicity analysis, both CP-even and CP-odd contributions are subsumed in "$g \times u$" term of Eq. (7), so that a population asymmetry in this case can exist without direct CP-violation. Thus for pairs with two spin one particles, a helicity analysis is required; similar considerations apply to pairs for which the decays are not quasi-two-body in nature—an ancillary angular analysis is necessary.

The observation of direct CP-violation in B-meson decays in itself is crucial to establishing the mechanism of CP-violation. Nevertheless, we would also like to interpret such results in terms of the parameters of the CKM matrix. An assumption of isospin symmetry can codify and potentially determine the hadronic parameters needed to interpret the mixing-induced CP-asymmetry in $b \to dq\bar{q}$ transitions to charmless final states. Relevant to the modes we discuss are the isospin-based analyses which yield $\sin(2\alpha)$ in $B \to \rho\pi$ [13,29,30] and $B \to a_1\pi$ [31] decays. These analyses, however, do not determine the parameters

[4] Recall that $\epsilon_{\rm eff}$, a conflation of the tagging efficiency ϵ and the mistag fraction w given by $\epsilon_{\rm eff} = \epsilon(1-2w)^2$, drives the statistical error in an asymmetry measurement as per $1/\sqrt{\epsilon_{\rm eff} N}$, where N is the number of untagged events.

Table 1
B_d decays to certain three-, four-, and five-particle, self-conjugate final-states and some of the CP-enantiomers they contain

3-particles	CP-enantiomers
$\pi^+\pi^-\pi^0$	$(\rho^+\pi^-, \rho^-\pi^+)$
$K^+K^-\pi^0$	$(K^*(892)^+K^-, K^*(892)^-K^+)$
$D^+D^-\pi^0$	$(D^*(2010)^+D^-, D^*(2010)^-D^+)$
$D^0\overline{D}^0\pi^0$	$(D^*(2007)^0\overline{D}^0, \overline{D}^*(2007)^0D^0)$
4-particles	**CP-enantiomers**
$\pi^+\pi^-\pi^0\pi^0$	$(\rho^+\pi^-\pi^0, \rho^-\pi^+\pi^0)^{\mathrm{a}}$
$\pi^+\pi^-\pi^+\pi^-$	$(a_1(1260)^+\pi^-, a_1(1260)^-\pi^+)$
$K^+K^-\pi^+\pi^-$	$(K^*(892)^0K^-\pi^+, \overline{K}^*(892)^0K^+\pi^-)^{\mathrm{a}}$
$D^0\overline{D}^0\pi^+\pi^-$	$(D^*(2010)^+\overline{D}^0\pi^-, D^*(2010)^-D^0\pi^+)^{\mathrm{a}}$
5-particles	**CP-enantiomers**
$\pi^+\pi^-\pi^+\pi^-\pi^0$	$(\rho^+\pi^-\pi^+\pi^-, \rho^-\pi^-\pi^+\pi^+)^{\mathrm{a}}$
	$(a_1(1260)^+\pi^-\pi^0, a_1(1260)^-\pi^+\pi^0)^{\mathrm{a}}$
	$(a_1(1260)^+\rho^-, a_1(1260)^-\rho^+)^{\mathrm{a}}$
	$(a_0(980)^+\pi^-, a_0(980)^-\pi^+)$
	$(b_1(1235)^+\pi^-, b_1(1235)^-\pi^+)$

[a] A helicity and/or angular analysis is required; see text.

necessary to interpret direct CP-violation; the terms containing $\sin\alpha$ and $\cos\alpha$ are multiplied by unknown hadronic parameters. Nevertheless, were $\sin(2\alpha)$ determined and direct CP-violation observed, the SM value of $\sin\alpha$ could be inferred, modulo discrete ambiguities. Interpreting direct CP-violating observables directly in terms of the underlying weak parameters may not prove possible. Theoretical progress has been made, however, in the computation of partial-rate asymmetries in some two-body decays, see, e.g., Refs. [32,33]. Alternatively, more phenomenological treatments indicate that the presence of resonances in certain channels can enhance the associated partial rate asymmetry [34,35] and aid in the extraction of weak phase information [36].

We have discussed the conditions under which the rich resonance structure of hadronic B decays can be exploited to search for direct CP-violation in untagged decays. Our method is sufficiently general to enable direct CP-violation searches in B_s and D meson decays as well. In some channels the untagged search we propose complements tagged, time-dependent analyses in $B \to \rho\pi$ and $B \to a_1\pi$ decays. Nevertheless, the gain in statistical power realized in untagged versus tagged searches, i.e., roughly a factor of 2 at the B-factories and of 4 in a hadronic environment such as at CDF, argues for a more comprehensive program.

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References

[1] KTeV Collaboration, A. Alavi-Harati, et al., Phys. Rev. Lett. 83 (1999) 22;
NA48 Collaboration, V. Fanti, et al., Phys. Lett. B 465 (1999) 335;
NA31 Collaboration, G.D. Barr, et al., Phys. Lett. B 317 (1993) 233.
[2] S. Bosch, et al., Nucl. Phys. B 565 (2000) 3.
[3] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531;
M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[4] BaBar Collaboration, B. Aubert, et al., Phys. Rev. Lett. 87 (2001) 091801;
Belle Collaboration, K. Abe, et al., Phys. Rev. Lett. 87 (2001) 091802.
[5] See, e.g., Y. Nir, hep-ph/0109090.
[6] M. Bander, D. Silverman, A. Soni, Phys. Rev. Lett. 43 (1979) 242.
[7] L. Wolfenstein, Phys. Rev. Lett. 13 (1964) 562.
[8] L. Wolfenstein, The superweak theory 35 years later, in: J.L. Rosner, B.D. Winstein (Eds.), Kaon Physics, Univ. of Chicago Press, Chicago, IL, 2001.
[9] A.B. Carter, A.I. Sanda, Phys. Rev. Lett. 45 (1980) 952;
A.B. Carter, A.I. Sanda, Phys. Rev. D 23 (1981) 1567.
[10] I.I. Bigi, A.I. Sanda, Nucl. Phys. B 193 (1981) 85;
I.I. Bigi, A.I. Sanda, Nucl. Phys. B 281 (1987) 41.
[11] I. Dunietz, J.L. Rosner, Phys. Rev. D 34 (1986) 1404.
[12] J.D. Roberts, M.C. Caserio, Basic Principles in Organic Chemistry, 2nd Edition, Benjamin, Menlo Park, CA, 1977.
[13] H.R. Quinn, J.P. Silva, Phys. Rev. D 62 (2000) 054002.
[14] S. Gardner, U.-G. Meißner, Phys. Rev. D 65 (2002) 094004;
J. Tandean, S. Gardner, Phys. Rev. D 66 (2002) 034019.
[15] Y.I. Azimov, N.G. Uraltsev, V.A. Khoze, Yad. Fiz. 45 (1987) 1412.

[16] BaBar Collaboration, B. Aubert, et al., hep-ex/0107058.
[17] S. Gardner, J. Tandean, in preparation.
[18] See, e.g., A. Ali, G. Kramer, C.D. Lu, Phys. Rev. D 58 (1998) 094009.
[19] N. Sinha, R. Sinha, Phys. Rev. Lett. 80 (1998) 3706.
[20] A.E. Snyder, private communication.
[21] I. Dunietz, et al., Phys. Rev. D 43 (1991) 2193;
A.S. Dighe, et al., Phys. Lett. B 369 (1996) 144.
[22] CLEO Collaboration, C.P. Jessop, et al., Phys. Rev. Lett. 79 (1997) 4533.
[23] G. Valencia, Phys. Rev. D 39 (1989) 3339.
[24] K. Anikeev, et al., hep-ph/0201071.
[25] P. Ball, et al., hep-ph/0003238.
[26] CDF Collaboration, T. Affolder, et al., Phys. Rev. D 61 (2000) 072005.
[27] BaBar Collaboration, B. Aubert, et al., Phys. Rev. D 66 (2002) 032003.
[28] Belle Collaboration, K. Abe, et al., Phys. Rev. D 66 (2002) 032007.
[29] H.J. Lipkin, Y. Nir, H.R. Quinn, A. Snyder, Phys. Rev. D 44 (1991) 1454.
[30] A.E. Snyder, H.R. Quinn, Phys. Rev. D 48 (1993) 2139.
[31] BaBar Collaboration, P.F. Harrison, H.R. Quinn (Eds.), The BaBar Physics Book: Physics at an Asymmetric B-Factory, SLAC-R-0504.
[32] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914;
M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 591 (2000) 313;
M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 606 (2001) 245.
[33] H.-n. Li, H.L. Yu, Phys. Rev. Lett. 74 (1995) 4388;
H.-n. Li, H.L. Yu, Phys. Rev. D 53 (1996) 2480;
Y.Y. Keum, H.-n. Li, A.I. Sanda, Phys. Rev. D 63 (2001) 054008.
[34] D. Atwood, A. Soni, Z. Phys. C 64 (1994) 241;
D. Atwood, A. Soni, Phys. Rev. Lett. 74 (1995) 220;
D. Atwood, G. Eilam, M. Gronau, A. Soni, Phys. Lett. B 341 (1995) 372;
G. Eilam, M. Gronau, R.R. Mendel, Phys. Rev. Lett. 74 (1995) 4984.
[35] R. Enomoto, M. Tanabashi, Phys. Lett. B 386 (1996) 413.
[36] S. Gardner, H.B. O’Connell, A.W. Thomas, Phys. Rev. Lett. 80 (1998) 1834.